Composite Bus Handling

External areas that are run on Composite Bus are handled by equivalently reducing the external area's injection onto the boundary buses between OPF areas and the external area. The equivalent reduction preserves the electrical dispatch, assuming transmission lines in the external area are *not* monitored. The following is an overview of the equivalent reduction methodology.

Let's assume there are two areas in a system, A_1 and A_2. Let there be n_1 buses in A_1 indexed $1, \ldots, n_1$, n_b boundary buses indexed $n_1 + 1, \ldots, n_1 + n_b$ between A_1 and A_2, and n_2 buses in A_2 indexed $n_1 + n_b + 1, \ldots, n_1 + n_b + n_2$.

When both A_1 and A_2 are running on OPF, the power flow equations are given by an equality constraint of the OPF formulation. Specifically, assuming transmission limits are ignored, it is given by

$$Y_{bus}\theta = P$$

where $P = G - D$, the vector of injection at each bus. Using the subscripts 1, B and 2 to denote buses in A_1, the boundary and A_2 respectively, one can partition the set of power flow equations for the network as:

$$\begin{bmatrix} Y_{11} & Y_{1B} & Y_{12} \\ Y_{1B} & Y_{BB} & Y_{B2} \\ Y_{12} & Y_{B2} & Y_{22} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_B \\ \theta_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_B \\ P_2 \end{bmatrix} \tag{1}$$

where, we have used the subscripts $Y_{i,j}$ to denote the sub-matrix of Y_{bus} indexed by the buses in the corresponding area set.

Given the described network structure we note that $Y_{12} = 0$, since all connections from A_1 to A_2 must traverse through at least one of the boundary bus set. This observation allows us to eliminate the θ_2 variables as follows.

$$\begin{bmatrix} Y_{11} & Y_{1B} \\ Y_{1B} & Y_{BB} - Y_{B2}Y_{22}^{-1}Y_{B2} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_B \end{bmatrix} = \begin{bmatrix} P_1 \\ P_B - Y_{B2}Y_{22}^{-1}P_2 \end{bmatrix} \tag{2}$$

The term $Y_{B2}Y_{22}^{-1}P_2$ maps the injection (Generation - Demand) at each bus in A_2 onto one or more boundary buses, while the term $Y_{B2}Y_{22}^{-1}Y_{B2}$ modifies the connections withing the boundary buses to effectively achieve an equivalent set of the flow. Notice that both factors are derived directly from the entries of the Y_{bus}, i.e. factors are governed solely by the underlying network.

$$\widehat{Y}_{bus} = \begin{bmatrix} Y_{11} & Y_{1B} \\ Y_{1B} & Y_{BB} - Y_{B2}Y_{22}^{-1}Y_{B2} \end{bmatrix}$$

The equivalenced matrix effectively defines a modified Y_{bus} matrix, denoted as $\widehat{Y}_{bus}$, that guarantees that the set of bus angles θ_1 and θ_B are the same value in Equations 1 and 2, and thereby preserving the powerflow in the external area, assuming transmission lines in the external are *not* monitored.